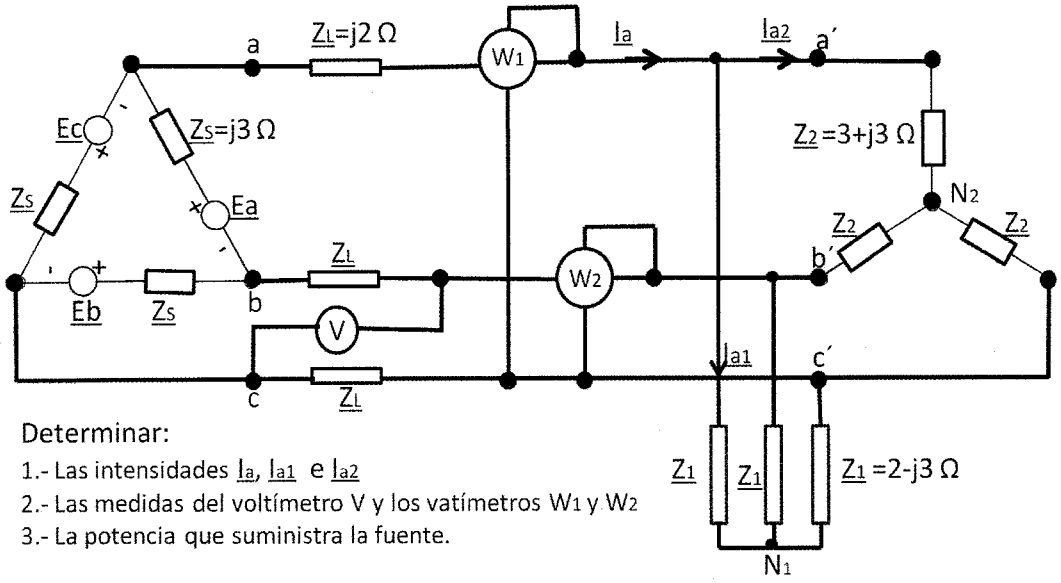
Ejercicio 191216

En el esquema de la figura inferior, los valores de las fem son: $\underline{Ea}$ = 207,846/0º, $\underline{Eb}$ = 207,846/-120º y $\underline{Ec}$ = 207,846/120º. Las impedancias tienen los siguientes valores: $\underline{Zs}$ = j3 Ω, $\underline{Z_L}$ = j2 Ω, $\underline{Z_1}$ =2- j3 Ω y y $\underline{Z_2}$ =3+j3.



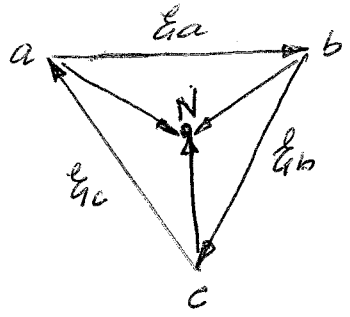
Determinar:

1.- Las intensidades $\underline{Ia}$, $\underline{Ia1}$ e $\underline{Ia2}$

2.- Las medidas del voltímetro V y los vatímetros W1 y W2

3.- La potencia que suministra la fuente.

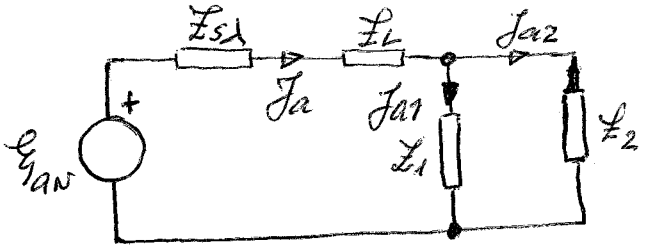
1) Sistema equilibrado $\Rightarrow N_1 = N_2 = N$



$$\mathcal{E}_{aN} = \frac{Ea}{\sqrt{3}}\angle -30^\circ = 120\angle -30^\circ$$

$$\mathcal{Z}_{s\lambda} = \frac{\mathcal{Z}_{s\Delta}}{3} = \frac{j3}{3} = j1\,\Omega$$

$$\mathcal{Z}_p = \frac{\mathcal{Z}_1 \mathcal{Z}_2}{\mathcal{Z}_1 + \mathcal{Z}_2} = \frac{(2-j3)(3+j3)}{2-3j+3+3j} = \frac{15-j3}{5} = 3-j0{,}6$$



$$\mathcal{Z}_{eq} = \mathcal{Z}_{s\lambda} + \mathcal{Z}_L + \mathcal{Z}_p =$$
$$= j1 + j2 + 3 - j0{,}6 = 3 + j2{,}4$$
$$= 3{,}84\angle 38{,}66$$

$$\mathcal{I}a = \frac{\mathcal{E}_{aN}}{\mathcal{Z}_{eq}} = \frac{120\angle -30^\circ}{3{,}84\angle 38{,}66} = 31{,}25\angle -68{,}66^\circ\ A$$

$$\begin{cases} \mathcal{I}_b = 31{,}25\angle -188{,}66 \\ \mathcal{I}_c = 31{,}25\angle 51{,}34^\circ A \end{cases}$$

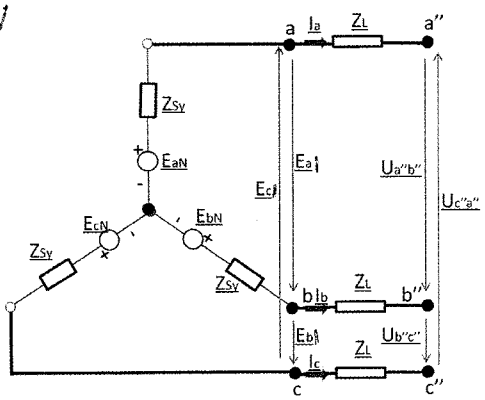
$$\mathcal{I}a_1 = \frac{\mathcal{I}a\,\mathcal{Z}_2}{\mathcal{Z}_1 + \mathcal{Z}_2} = \frac{31{,}25\angle -68{,}66\,(3+j3)}{5} = \frac{31{,}25\angle -68{,}66 \cdot 4{,}24\angle 45^\circ}{5}$$

$$\mathcal{I}a_1 = 26{,}5\angle -23{,}66^\circ A$$

$$\mathcal{I}a_2 = \frac{\mathcal{I}a\,\mathcal{Z}_1}{\mathcal{Z}_1 + \mathcal{Z}_2} = \frac{31{,}25\angle -68{,}66\,(2-j3)}{5} = \frac{31{,}25\angle -68{,}66 \cdot 3{,}61\angle -56{,}31^\circ}{5}$$

$$\mathcal{I}a_2 = 22{,}56\angle -124{,}97^\circ A$$

2)



$W_1 = W_a(a''c'') = real\{U_{a''c''}\,\underline{I}_a^*\}$

$W_2 = W_b(b''c'') = real\{U_{b''c''}\,\underline{I}_b^*\}$

$\underline{I}_a = 31{,}25\angle -68{,}66$ $\quad \underline{E}_{aN} = 120\angle -30^\circ$ V

$\underline{I}_b = 31{,}25\angle 171{,}34$ $\quad \underline{E}_{bN} = 120\angle -150^\circ$ V

$\underline{I}_c = 31{,}25\angle 51{,}34$ $\quad \underline{E}_{cN} = 120\angle 90^\circ$ V

$\underline{E}_{a1} - \underline{Z}_{SY}\underline{I}_b + \underline{E}_{bN} - \underline{E}_{aN} + \underline{Z}_{SY}\underline{I}_a = 0$

$\underline{E}_{a1} - 1\angle 90 \cdot 31{,}25\angle 171{,}34 + 120\angle -150 - 120\angle -30 + 1\angle 90 \cdot 31{,}25\angle -68{,}66 = 0$

$\underline{E}_{a1} - 31{,}25\angle -98{,}66 + 120\angle -150 - 120\angle -30 + 31{,}25\angle 21{,}34 = 0$

$\underline{E}_{a1} - (-5{,}244 - j30{,}894) + (-103{,}92 - j0{,}5) - (103{,}92 - j0{,}5) + (29{,}11 + j11{,}372) = 0$

$\underline{E}_{a1} + (5{,}244 - 103{,}92 - 103{,}92 + 29{,}11) + j(30{,}894 + j11{,}372) = 0$

$\underline{E}_{a1} - 173{,}486 + j42{,}266 = 0 \quad \underline{E}_{a1} = 173{,}486 - j42{,}266 = 178{,}56\angle -13{,}69^\circ$

$\underline{E}_{b1} = 178{,}56\angle -133{,}69^\circ \quad \underline{E}_{c1} = 178{,}56\angle 106{,}31^\circ$

$$\left.\begin{aligned} \underline{E}_{c1} + \underline{Z}_L\underline{I}_a - U_{c''a''} - \underline{Z}_L\underline{I}_c = 0 \\ \underline{E}_{b1} + \underline{Z}_L\underline{I}_c - U_{b''c''} - \underline{Z}_L\underline{I}_b = 0 \end{aligned}\right\} \Rightarrow U_{a''c''} = -U_{c''a''} = \underline{Z}_L\underline{I}_c - \underline{Z}_L\underline{I}_a - \underline{E}_{cA}$$

$U_{a''c''} = 2\angle 90 \cdot 31{,}25\angle 51{,}34 - 2\angle 90\ 31{,}25\angle -68{,}66 - 178{,}56\angle 106{,}31 =$

$= -48{,}80 + j39{,}04 - 58{,}21 - j22{,}74 + 50{,}146 - j171{,}374 =$

$= -56{,}864 - j155{,}074 = 165{,}17\angle -110{,}137^\circ$ V

$\underline{I}_a = 31{,}25\angle -68{,}66 = 11{,}37 - j29{,}11 \quad \underline{I}_a^* = 11{,}37 + j29{,}11 = 31{,}25\angle 68{,}66$

$\underline{S}_1 = 165{,}17\angle -110{,}137 \cdot 31{,}25\angle 68{,}66 = 5161{,}56\angle -41{,}477 = 3.867{,}15 + j\,3.418{,}60$

$U_{b''c''} = \underline{E}_{b1} + \underline{Z}_L\underline{I}_c - \underline{Z}_L\underline{I}_b = 178{,}56\angle -133{,}69 + 2\angle 90 \cdot 31{,}25\angle 51{,}34 - 2\angle 90\ 31{,}25\angle -188{,}66$

$= 178{,}56\angle -133{,}69 + 62{,}50\angle 141{,}34 - 62{,}50\angle -98{,}66$

$= -123{,}34 - j129{,}11 - 48{,}80 + j39{,}04 + 9{,}41 + j61{,}79 = -162{,}73 - j28{,}28$

$= 165{,}17\angle -170{,}14^\circ$ V

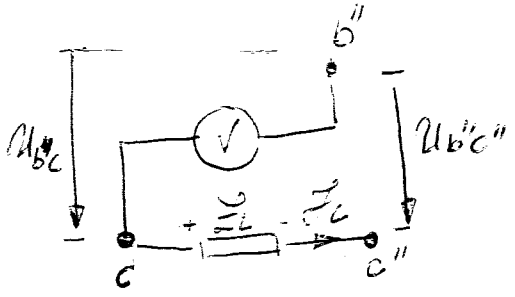
$\underline{I}_b = 31{,}25\angle -188{,}66 = -30{,}89 + j4{,}71 \quad \underline{I}_b^* = -30{,}89 - j4{,}71 = 31{,}25\angle -171{,}34^\circ$

$\underline{S}_2 = U_{b''c''}\,\underline{I}_b^* = 165{,}17\angle -170{,}14 \cdot 31{,}25\angle -171{,}34 = 5161{,}56\angle 18{,}52$

$= 4.894{,}26 + j\,1639{,}50$

$W_1 = 3.867{,}15$ W $\qquad W_2 = 4.894{,}26$ W

$P_\Sigma = W_1 + W_2 = 3.867{,}15 + 4.894{,}26 = 8.761{,}41$ W



$$\dot{U}_{b''c} + \dot{E}_L - \dot{U}_{b''c''} = 0$$

$$\dot{U}_{b''c} = \dot{U}_{b''c''} - \dot{E}_L = \dot{I}_c =$$

$$= 165,17 \angle -170,14 - 2 \angle 90 \cdot 31,25 \angle 51,34 =$$

$$= 165,17 \angle -170,14 - 62,50 \angle 141,34 = -162,73 - j28,28 + 48,80 - j39,04$$

$$= -113,93 - j67,32 = 132,33 \angle -149,42 = V$$

3) $\quad 894,26 - 3867,15 = -\frac{Q_c}{\sqrt{3}} \Rightarrow Q_c = -1779 \text{ VAr}$

$$Q_L = 3 \cdot j2 \cdot 31,25^2 = 5.859,37 \text{ VAr} \qquad P_L = 0$$

$$Q_g = Q_L + Q_c = 5.859,37 - 1779 = 4080,37 \text{ VAr}$$

COMPROBACIÓN:

$$\dot{E}_{a1} = 178,56 \angle -13,69 \text{ V}$$

$$\dot{E}_{a1N} = \frac{178,56}{\sqrt{3}} \angle -13,69 - 30 = 103,09 \angle -43,90$$

$$\dot{I}_a = 31,25 \angle -68,66 \Rightarrow \dot{I}_a^* = 31,25 \angle 68,66 \text{ A}$$

$$\dot{S}_g = 3 \dot{E}_{a1N} \cdot \dot{I}_a^* = 3 \cdot 103,09 \angle -43,90 \cdot 31,25 \angle 68,66 =$$

$$= 9.664,69 \angle 24,76 = 8.776,21 + j4.047,74 = P_g + jQ_g$$

$$S_{c1} = 3 \dot{Z}_1 I_{a1}^2 = 3(2 - j3) \cdot 26,5^2 = 4.213,5 - j6.320,25$$

$$S_{c2} = 3 \dot{Z}_2 I_{a2}^2 = 3(3 + j3) \cdot 22,56^2 = 4.580,58 + j4580,58$$

$$W_1 + W_2 = 4213,5 + 4.580,58 = 8.794,08 \text{ W}$$

$$W_1 - W_2 = \frac{Q_c}{\sqrt{3}} = \frac{-6.320,25 + 4.580,58}{\sqrt{3}} = -1004,40$$

$$2W_1 = 7.789,68 \text{ W} \rightarrow W_1 = 3.894,84 \text{ W}$$

$$W_2 = W_1 + 1.004,40 = 4.899,24 \text{ W}$$

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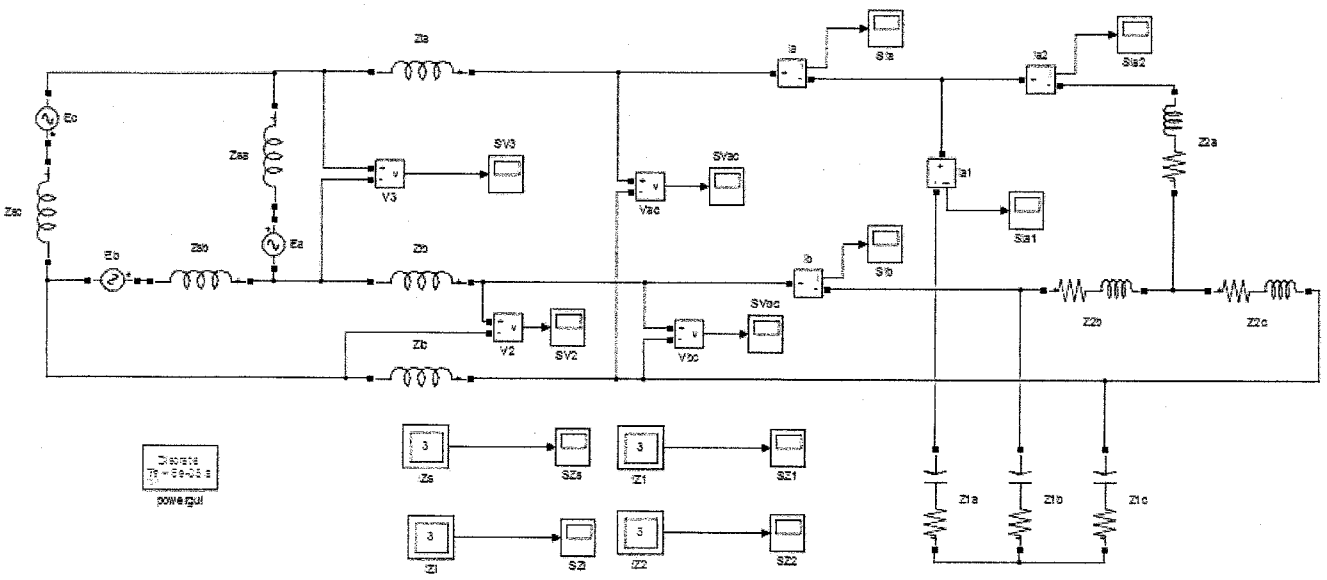


Steady state values:

```
MEASUREMENTS:

  1:   'U Vbc  '  =        165.49 Vrms  -169.9°
  2:   'U Vac  '  =        165.49 Vrms  -109.9°
  3:   'U V3   '  =        179.10 Vrms   -13.6°
  4:   'U V2   '  =        132.80 Vrms  -149.2°
  5:   'I Ib   '  =         31.23 Arms   171.3°
  6:   'I Ia   '  =         31.23 Arms   -68.7°
  7:   'I Ia1  '  =         26.50 Arms   -23.7°
  8:   'I Ia2  '  =         22.52 Arms  -124.9°
  9:   'Ub: Z1a'  =         95.54 Vrms   100.0°
 10:   'Ub: Z1b'  =         95.54 Vrms   -19.9°
 11:   'Ub: Z1c'  =         95.54 Vrms  -139.9°
 12:   'Ub: Z2a'  =         95.54 Vrms   100.0°
 13:   'Ub: Z2b'  =         95.54 Vrms   160.0°
 14:   'Ub: Z2c'  =         95.54 Vrms  -139.9°
 15:   'Ib: Z1a'  =         26.50 Arms   156.3°
 16:   'Ib: Z1b'  =         26.50 Arms    36.3°
 17:   'Ib: Z1c'  =         26.50 Arms   -83.7°
 18:   'Ib: Z2a'  =         22.52 Arms    55.0°
 19:   'Ib: Z2b'  =         22.52 Arms   115.0°
 20:   'Ib: Z2c'  =         22.52 Arms   175.0°
 21:   'Ib: Z1a'  =         31.23 Arms   111.3°
 22:   'Ib: Z1b'  =         31.23 Arms    -8.7°
 23:   'Ib: Z1c'  =         31.23 Arms  -128.7°
 24:   'Ib: Zsa'  =         18.03 Arms   141.3°
 25:   'Ib: Zsb'  =         18.03 Arms    21.3°
 26:   'Ib: Zsc'  =         18.03 Arms   -81.3°
```

Units:

RMS values

Frequency:

50

Display:

- [ ] States
- [x] Measurements
- [ ] Sources
- [ ] Nonlinear elemen

Format:

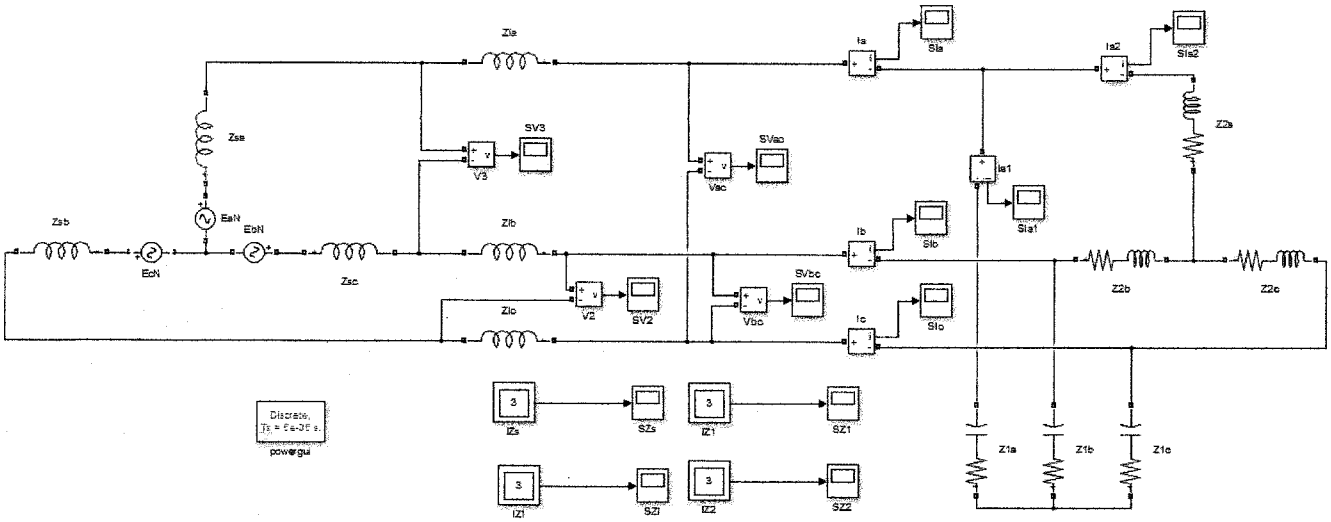
2590571.12

Ordering:

Name then value

Update Steady State

MATLAB/SIMULINK Ej. 191216



Steady state values:

```
MEASUREMENTS:

  1:   'U Vbc   '  =        165.49 Vrms   -169.98°
  2:   'U Vac   '  =        165.49 Vrms   -109.98°
  3:   'U V3    '  =        179.10 Vrms    -13.64°
  4:   'U V2    '  =        132.81 Vrms   -149.28°
  5:   'I Ic    '  =         31.23 Arms     51.33°
  6:   'I Ib    '  =         31.23 Arms    171.33°
  7:   'I Ia2   '  =         22.52 Arms   -124.98°
  8:   'I Ia1   '  =         26.50 Arms    -23.67°
  9:   'I Ia    '  =         31.23 Arms    -68.67°
 10:   'Ub: Z1a'  =         95.54 Vrms    100.02°
 11:   'Ub: Z1b'  =         95.54 Vrms    -19.98°
 12:   'Ub: Z1c'  =         95.54 Vrms   -139.98°
 13:   'Ub: Z2a'  =         95.54 Vrms    100.02°
 14:   'Ub: Z2b'  =         95.54 Vrms    160.02°
 15:   'Ub: Z2c'  =         95.54 Vrms   -139.98°
 16:   'Ib: Z1a'  =         26.50 Arms    156.33°
 17:   'Ib: Z1b'  =         26.50 Arms     36.33°
 18:   'Ib: Z1c'  =         26.50 Arms    -83.67°
 19:   'Ib: Z2a'  =         22.52 Arms     55.02°
 20:   'Ib: Z2b'  =         22.52 Arms    115.02°
 21:   'Ib: Z2c'  =         22.52 Arms    175.02°
 22:   'Ib: Zla'  =         31.23 Arms    111.33°
 23:   'Ib: Zlb'  =         31.23 Arms     -8.67°
 24:   'Ib: Zlc'  =         31.23 Arms   -128.67°
 25:   'Ib: Zsa'  =         31.23 Arms    -68.67°
 26:   'Ib: Zsb'  =         31.23 Arms     51.33°
 27:   'Ib: Zsc'  =         31.23 Arms    171.33°
```

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RMS values

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50

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Update Steady State Va