Ejercicio 12-2

1.- En el sistema de la figura, calcular las intensidades de línea y las tensiones de fase en la carga, para los siguientes datos:

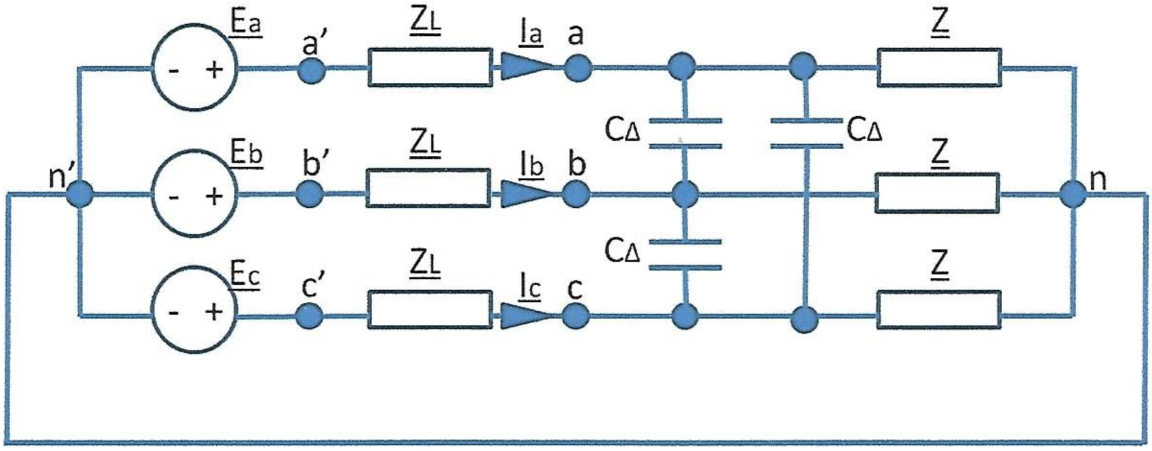
Tensiones de fase en el generador ideal:

$\underline{E_a} = 220\angle 0º$ V, $\underline{E_b} = 220\angle -120º$ V, $\underline{E_c} = 220\angle 120º$ V

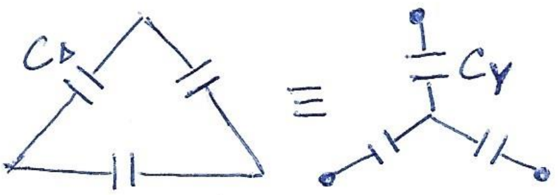
Impedancia de cada conductor de línea: $\underline{Z_L} = 1+j0\ \Omega$

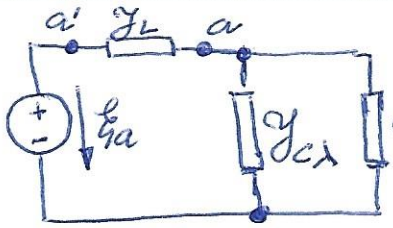
Impedancia de cada fase de la carga en estrella: $\underline{Z} = 8+j6\ \Omega$

Admitancia de cada fase de los condensadores en triángulo: $\underline{Y_{C\Delta}} = j0{,}02$ S



2.- Suponer que $\underline{E_a} = 0$ V y que el resto de los datos se mantiene y calcular de nuevo las intensidades de línea y las tensiones de fase en la carga.





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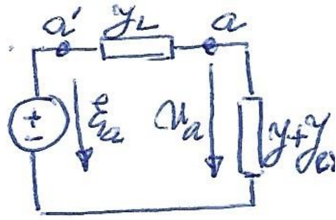


$Y_{C\Delta} = j0{,}02 \quad Y_{CY} = 3Y_{C\Delta} = j0{,}06\,S \quad Y_L = 1\,S \quad Y = \frac{1}{Z} = \frac{1}{8+j6} = \frac{1}{10\angle 36{,}87} = 0{,}08 - j0{,}06$

$Y_L + Y_{CY} + Y = 1 + 0{,}08 = 1{,}08\,S \qquad Y_{CY} + Y = j0{,}06 + 0{,}08 - j0{,}06 = 0{,}08$

$$U_a = \frac{Y_L . E_a}{Y_L + Y_{CY} + Y} = \frac{220\angle 0°}{1{,}08} = 203{,}70\angle 0°$$

$$U_c = 203{,}70\angle +120°$$

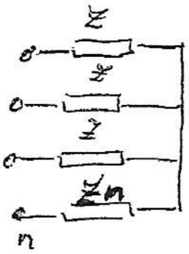
$$U_b = \frac{Y_L E_b}{Y_L + Y_{CY} + Y} = \frac{220\angle -120°}{1{,}08} = 203{,}70\angle -120°$$

$$I_a = Y_L(E_a - U_a) = (Y_{CY} + Y)U_a = 0{,}08 \cdot 203{,}70\angle 0° = 16{,}296\angle 0°$$

$$I_b = Y_L(E_b - U_b) = (Y_{CY} + Y)U_b = 0{,}08 \cdot 203{,}70\angle -120° = 16{,}296\angle -120°$$

$$I_c = Y_L(E_c - U_b) = (Y_{CY} + Y)U_c = 16{,}296\angle 120°$$

Teorema de Millman $\Rightarrow$ $V_{n'n} = \dfrac{\sum_{k=1}^{n} V_k \cdot \frac{1}{Z_k}}{\sum_{i=1}^{n} \frac{1}{Z_i}}$



(*)

$\Rightarrow Y_a = Y_b = Y_c = \quad = 1{,}08$

$$U_{n''n'} = \frac{E_a Y_a + E_b Y_b + E_c Y_c}{Y_a + Y_b + Y_c + Y_n} = \frac{Y_\lambda (E_a + E_b + E_c)}{3 Y_\lambda + Y_n} \quad (2)$$

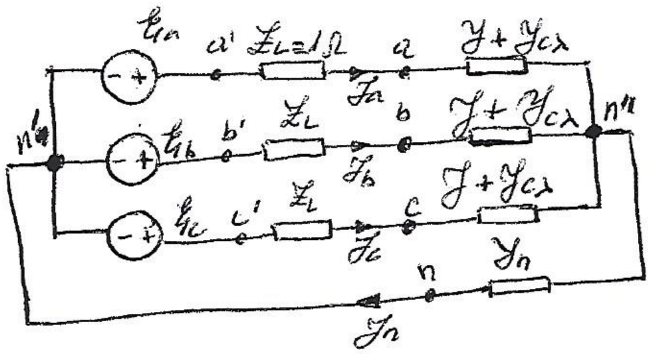
$$(*) \quad Z_n = \frac{1}{3} \frac{Z_1^2}{Z_1 + Z_2} \Rightarrow Y_n = \frac{3(Z_1 + Z_2)}{Z_1^2} = \frac{3\left(\frac{1}{Y_1} + \frac{1}{Y_2}\right)}{\left(\frac{1}{Y_1}\right)^2} = \frac{3\left(\frac{Y_1 + Y_2}{Y_1 \cdot Y_2}\right)}{\left(\frac{1}{Y_1}\right)^2} \Rightarrow$$

$$Y_n = 3\left(\frac{Y_1 + Y_2}{Y_1 \cdot Y_2}\right) Y_1^2 = 3 \frac{(Y_1 + Y_2) Y_1}{Y_2}$$

$$\begin{cases} Y_1 + Y_2 = Y + Y_{c\lambda} = 0{,}08\ s \\ Y_1 = \frac{1}{8 + j6} = Y \quad Y_2 = Y_{c\lambda} = 0{,}06\ s \end{cases}$$

$$= \frac{3 \cdot 0{,}08 \cdot \left(\frac{1}{8 + j6}\right)}{j0{,}06} = \frac{0{,}24}{j0{,}06(8 + j6)} = \frac{0{,}24}{j0{,}48 - 0{,}36} = \frac{-0{,}24}{(0{,}36 - j0{,}48)} \Rightarrow$$

$$Y_n = \frac{-0{,}24(0{,}36 + j0{,}48)}{(0{,}36 - j0{,}48)(0{,}36 + j0{,}48)} = \frac{-0{,}24(0{,}36 + j0{,}48)}{0{,}36} = \underline{\underline{-0{,}24 - j0{,}32}}$$



$$Y_\lambda = \frac{(Y + Y_{c\lambda}) \cdot Y_L}{Y + Y_{c\lambda} + Y_L} = (1)$$

$$\begin{cases} Y + Y_{c\lambda} + Y_L = 1{,}08\ s \\ Y + Y_{c\lambda} = 0{,}08\ s \\ Y_L = 1\ s \end{cases}$$

$$Y = \frac{1}{Z} = \frac{1}{8 + j6} = \frac{8 - j6}{64 + 36} = 0{,}08 - j0{,}06$$

$$Y + Y_{c\lambda} = 0{,}08 - j0{,}06 + j0{,}06 = 0{,}08$$

$$(1) \quad Y_\lambda = \frac{0{,}08 \cdot 1}{1{,}08} = 0{,}074\ s$$

$$(2) \quad U_{n''n'} = \frac{0{,}074(0 + 220\angle{-120} + 220\angle{120})}{3 \cdot 0{,}074 - 0{,}24 - j0{,}32} = \frac{0{,}074 \cdot 2 \cdot 220 \cos 120}{-(0{,}018 + j0{,}32)} =$$

$$= \frac{0{,}074 \cdot 2 \cdot 220 \cdot \cos 120 (0{,}018 - j0{,}32)}{-(0{,}018 + j0{,}32)(0{,}018 - j0{,}32)} = \frac{16{,}28(0{,}018 - j0{,}32)}{0{,}1027} =$$

$$= 158{,}48(0{,}018 - j0{,}32) = 2{,}85 - j50{,}71 = \underline{50{,}79\angle{-86{,}78}}$$

$$\mathcal{I}_a = \mathcal{Y}_Y\,(\overset{0}{\mathcal{E}_a} - U_{n''n'}) = -\,\mathcal{Y}_Y \cdot U_{n''n'} = -0{,}074\,(2{,}85 - j50{,}71) = -0{,}2109 + j3{,}7525$$
$$= \underline{\underline{3{,}758 \angle 93{,}22^\circ}}$$

$$\mathcal{I}_b = \mathcal{Y}_Y\,(\mathcal{E}_b - U_{n''n'}) = 0{,}074\,(220\angle{-120} - 50{,}79\angle{-86{,}78}) =$$
$$= 0{,}074\,[(220\cos(-120^\circ) + j220\,\mathrm{sen}(-120)) - (2{,}85 - j50{,}71)]$$
$$= 0{,}074\,[-110 - j190{,}52 - 2{,}85 + j50{,}71]$$
$$= -0{,}074\,(112{,}85 + j139{,}81) = (-8{,}35 - j10{,}346) = \underline{\underline{13{,}30\angle{-128{,}91}}}$$

$$\mathcal{I}_c = \mathcal{Y}_Y\,(\mathcal{E}_c - U_{n''n'}) = 0{,}074\,(220\angle{120} - 50{,}79\angle{-86{,}78}) =$$
$$= 0{,}074\,[(220\cos 120 + j220\,\mathrm{sen}\,120) - (2{,}85 - j50{,}71)]$$
$$= 0{,}074\,(-110 + j190{,}52 - 2{,}85 + j50{,}71) =$$
$$= 0{,}074\,(-112{,}85 + j241{,}23) = -8{,}35 + j17{,}85$$
$$= \underline{\underline{19{,}71\angle{115{,}07}}}$$

$$\mathcal{I}_n = \mathcal{I}_a + \mathcal{I}_b + \mathcal{I}_c = \mathcal{Y}_n \cdot U_{n''n'}$$
$$= (-0{,}24 - j0{,}32)(2{,}85 - j50{,}71) = -16{,}91 + j11{,}258 = \underline{\underline{20{,}32\angle{146{,}35}}}$$

El neutro n'' no es el neutro de la impedancia $\mathcal{Z}$ del circuito original. Las tensiones de fase en la carga $\mathcal{Z}$ son:

$$U_{an} = \frac{\mathcal{I}_a}{\mathcal{Y}_{CY} + \mathcal{Y}} + U_{n''n'} = -\frac{\mathcal{I}_a}{\mathcal{Y}_{L=1}} + \overset{0}{\mathcal{E}_a} = -\mathcal{I}_a = 0{,}2109 - j3{,}7525 = \underline{\underline{3{,}758\angle{-86{,}78^\circ}}}$$

$$U_{bn} = \frac{\mathcal{I}_b}{\mathcal{Y}_{CY} + \mathcal{Y}} + U_{n''n'} = -\frac{\mathcal{I}_b}{\mathcal{Y}_L} + \mathcal{E}_b = 8{,}35 + j10{,}346 + (-110 - j190{,}52)$$
$$= -101{,}65 - j180{,}174 = \underline{\underline{206{,}87\angle{-119{,}43^\circ}}}$$

$$U_{cn} = \frac{\mathcal{I}_c}{\mathcal{Y}_{CY} + \mathcal{Y}} + U_{n''n'} = -\frac{\mathcal{I}_c}{\mathcal{Y}_L} + \mathcal{E}_c = -8{,}35 + j17{,}85 + (-110 + j190{,}52)$$
$$= -118{,}35 + j208{,}37 = \underline{\underline{239{,}63\angle{119{,}60}}}$$